

Can the bar model support pupils to effectively and independently solve a range of words problems?

Orchard Primary School, Middle Phase

Abstract

The aim of this study was to find out if using bar models support pupils to solve a range of word problems effectively and independently. To trial its effectiveness, three low attaining pupils were selected from each class, from years three and four. Over a period of twelve weeks, pupils were taught a weekly twenty minute session on how to read and understand a problem using the bar model. The problems related to different areas of math, including the four operations and fractions. They ranged from simple single-step problems to complex multi-steps ones and were in line with the National Curriculum. In addition, Pupils practised using the skills modelled to solve similar problems independently. Gradually over time, there was less scaffolding and greater opportunities to apply using bar models more independently.

At the beginning, the participants showed no understanding of how to use the bar model. At the midpoint, 20 percent of pupils demonstrated clear understanding, more independence and fluency on how to use the bar model, whilst others still relied on adult support. Towards the end of the study, most pupils demonstrated their abilities to use the bar model independently and a clearer strategic approach to problem solving.

Introduction

Orchard Primary School is situated in the London Borough of Hackney, catering for pupils between the ages of three and eleven and is in the process of expanding from two forms of entry to three forms of entry. The pupils come from a wide range of nationalities and the majority are from minority ethnic backgrounds, most of them speak English as an additional language. At the end of Key Stage 2, pupils at Orchard are meeting the expected or higher standard in Reading, Writing and Maths is greater than the national average.

After a discussion around challenges pupils may encounter in numeracy, middle phase have concluded that word problems can often pose a challenge to pupils' learning as they require pupils to read the problem, visualise, identify key vocabulary and the question that needs to be answered, solve the problem using a method they are familiar with and finally explain their choices. Knowing how to calculate and understanding key vocabulary is not enough to solve words problems effectively. The main barriers to solving words problems in middle phase were that some children were not able to understand the problem in context, visualise it, know what to do systematically and explain their methods. Some pupils are not able to read it independently.

Bar models, which are used in Singapore Math, are pictorial representations of mathematical concepts, in particular rectangles. It supports the concrete, pictorial and abstract approach (CPA), developed by Bruner (1961). CPA is an effective approach to teaching that develops a deep understanding of Math, broadening pupils' understanding of challenging concepts.

The concrete method enables pupils to understand concepts using hands on components. This method is used a lot to teach younger pupils and the abstract method is more used to teach older pupils. The pictorial method is equally as important to pupils as it can make grasping challenging concepts easier. To solve word problems effectively, pupils will need visualisation. Bar models fit into the pictorial method, where pupils are able to visualise in their minds and represent concepts using pictures or diagrams. Once pupils have demonstrated a good understanding of this pictorial method, they can swiftly move onto using the abstract method to solve word problems.

Bar model is a "draw a picture" problem-solving strategy (Hoven & Garelick, 2007). It can enable pupils to work independently, by encouraging them to illustrate the problem. The bar model represents visually the information that pupils already know and what is unknown and this helps them to identify how that information can be used to solve the problem (Hoven & Garelick (2007). Furthermore, bar models are 'clearly designed to help pupils decide which operations to use' (Beckmann, 2004).

In the example (Appendix 1), the known information is that both children have 18 sweets in total and Sam has 12 of them. The unknown information is the number of sweets Sally has. The visual representation of this problem can then enable a pupil to realise that by subtracting 12 from 18, they will be able to find out how many sweets Sally has.

According to Hoven and Garelick (2007), bar models enable pupils to solve challenging word problems. It is a versatile method, which can be used by younger children to understand and solve simple word problems and can be adapted for older children to use when solving more complex ones related to areas such as: fractions, ratios and other word problems which involve more than one operation. For the use of bar models to be effective, it needs to be used consistently through the whole of Key Stage 1 and 2 and along with both concrete and abstract methods.

Research Process

The research process involved choosing three pupils from the bottom twenty percent from each class. Initially, pupils were presented with three word problems, related to addition and subtraction, which they had to solve independently and this was called the cold task.

After the cold task, the pupils worked with the class teacher once per week to solve three problems using the bar model. The first problem was to be carefully modelled by the class teacher (I do) and the second problem was to be solved as a group (we do), creating an additional opportunity for the adult to re-model or address any misconceptions. The third problem was an opportunity for pupils to apply the skills they have been modelled and independently solve the problem (you do).

This process continued for ten weeks and throughout it, the activities linked to different areas of math, such as: addition, subtraction, multiplication, division and some problems related to fractions and varied in levels of challenge.

Once per week, adults of middle phase dialogued around the project, where they had discussed things that have been successful as well areas for improvement. One

teacher suggested “The ‘we do’ and ‘you do’ questions should be worded similar to the ‘I do’ question as it will ensure the pupils confidently use the skills modelled by the adult immediately.” Another teacher shared that “the children should be taught how to draw their own bars if they are going to be using this method in future learning.”

The weekly discussions led to small changes in the project. Firstly, tasks gradually became more challenging. The ‘I do, we do, you do’ structure slowly became the ‘you do, you do, you do’ structure towards the end (Appendix 2). Furthermore, in the beginning, pupils had a bar already there for them to use and later on in the project, they had to draw their own. In the second half of the project, word problems became more multi-step and some involved the use of more than one operation.

After the ten weeks, the children were presented with a number of word problems to solve independently using the bar model and this was known as the hot task. The only support pupils were given in the hot task was support with reading the word problems. This task was an opportunity for pupils to apply the skill of bar modelling they had practised throughout the ten weeks and it was an opportunity for the adults to measure the progress from the cold task to the hot task.

Findings

In the cold task, many pupils demonstrated a good ability to read the problem and identify key vocabulary. However, more than half of the participants made incorrect links between the key vocabulary and the operations involved and those that did, made mistakes in their calculations. It was evident to see that one of the challenges to solving words problems for these pupils was the ability to correctly comprehend the meaning of particular words, which they may not be familiar with, such as ‘fewer’ or ‘sum’.

In Year Three, none of the participants visually represented the problems using any pictorial method and only some used an abstract method. For word problems to be effectively solved, they need to use method, which will enable visualisation. As there was no evidence of this, there were only two correct answers from a possible twenty-seven. In Year Four, all pupils attempted to represent the problems using the

abstract method (compact subtraction/addition method). However just as Year Three, there was no evidence of any pictorial method. The number of correct answers was greater than Year Three, although it was still only five from a possible eighteen.

After the cold task, outcomes for tasks one and two, shows pupils already making good use of the bar model, in particular year three. Pupils practised reading the problems, identified key vocabulary and used the bar to help them visualise, understand the problem better in context and make the correct links between the vocabulary and the operation involved.

After visually representing the word problems, most of the pupils recognised that the word problems involved multiplication and this enabled them to represent the problems symbolically. It was clear evident at this stage that the bar model can help pupils decide which operations to use. A small number of pupils, in particular year four, continued using the abstract method to solve the word problems and they did this successfully. As a result, tasks four and five were adapted to become even more challenging and encourage the use of the bar model.

In tasks four and five, many pupils in both year groups were beginning to demonstrate a good ability to solve words using the bar model as well as abstract methods they are familiar with. In order for pupils to become more independent, two changes were made. Firstly, the 'I do, we do, you do' structure became the 'I do, you do, you do' structure, which meant pupils would have a second opportunity to independently apply the skills modelled to them. Secondly, after a suggestion made by an adult, pupils now had to draw their own bar for each problem they had to solve (Appendix 2).

At this stage, twenty percent of the pupils were demonstrating a secure understanding of how to use the bar model to solve word problems (Appendix 4), with the majority being partially secure and a very small number of participants showing little or no understanding of how to use this method. When pupils were asked what they thought about this method, there was a mixture of views towards this model such as "It is easy to draw the bar and I can understand the problem better" (KA, Class 1) and "I find it difficult to draw the bar" (MA, Class 4).

Action research tasks eight to ten became the 'you do, you do, you do' structure after an example of solving a similar problem was modelled by the class teacher (Appendix 2) This was to enable pupils to become more independent in the application on this new skill. Participants in year three demonstrated excellent use of the bar model as well as an abstract method beside it and pupils in year four, who had not always used the pictorial method, did so more effectively. All pupils were ready to demonstrate complete independence in the hot task.

The hot task (Appendix 3) consisted of five 'you do' questions linked a range of different areas of math. Pupils were supported in reading the word problems, however, had to complete these independently. Eight out fifteen pupils demonstrated a solid understanding of solving the problems using the bar model, with almost all word problems solved correctly, demonstrating use of both the pictorial and the abstract method effectively and were able to explain their choices with ease.

The rest of the participants demonstrated some good understanding of using the bar model, correctly solving some word problems independently. However, not all problems were interpreted correctly and those which were, showed mistakes in calculations. A small number of pupils had to read the word problems again and check their answers. In addition, they were unable to represent the words problems using both the pictorial and abstract method.

Impact and Conclusion

By the end of this research project, more than half of the participants have been able to demonstrate secure knowledge and application of this pictorial strategy (Appendix 4). The rest were able to do this partially. All pupils' were able to demonstrate excellent use of this new skill during additional opportunities provided by their class teachers, such as whole maths class lessons and homework (Appendix 5).

The teachers in charge were very positive about the project and expressed a range of views from "This method really helped the children in breaking down the word problem in smaller small steps." to 'It helped create a clear picture of what the problem was about.'" However, one of the teachers expressed "The bar model

project would have been more successful if it was embedded from a younger age, such as Year 1.” Not all children were able to apply the use of bar models with fluency and ease as they were used to solving word problems with other pictorial representations such as drawing counters. This was evident in the early tasks completed by year four.

Furthermore, some teachers were more experienced at modelling how to use this method than others and this was evident to see in some of the pupils’ outcomes. One teacher expressed “It would be beneficial if all teachers were given training on how to teach challenging concepts using the bar model as part of a math inset,”

Looking at pupils’ outcomes, pupils were able use the bar model to visualise the problem, know what operation to use and some were able to represent the word problems using symbols. However, some of their calculations were not always accurate as they made mistakes such as forgetting to exchange when subtracting. Furthermore, when solving problems related to multiplication and division, some pupils were not able to recall multiples of numbers correctly and this influenced their final answers.

In addition, a group of pupils were not familiar with all the mathematical vocabulary involved. They were able to explain the most common vocabulary such as ‘total’ and ‘how many left’ with ease, however, they showed complexity in explaining phrases such as ‘five times as many’ or ‘how many where there to start with’, which meant certain key words were misinterpreted, leading to bar models being drawn incorrectly.

To summarise, the evidence from the research project concludes that bar models can enable pupils to independently understand, visualise a word problem and decide which operation to use when solving a word problem. All pupils can use this strategy, especially when they come across a complex word problem. In order for it to be more successful, bar models should consistently used throughout Key Stage 1 and 2. Pupils will need practise for fluency in calculating using taught methods and need richer opportunities to explore different mathematic vocabulary in various contexts.

Finally, teachers would greatly benefit from additional training on how to use this method correctly and effectively in their teaching.

References

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Bruner, J.S. (1961). Acts of Discovery. *Harvard Education Review*, 31 (1) p. 21-32.
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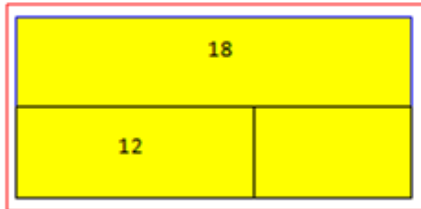
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Appendix 1

An example of a problem

Sam and Sally have 18 sweets in total.
If Sam has 12, how many does Sally have?



Appendix 2

The 'I do, we do, you' changing gradually to the 'you do, you do, you do' structure.

Year 3 Action Research - 1

I do: A shop employs 6 people to serve customers. Argos employs 4 times more.
How many people work at Argos? $6 \ 6 \ 6 \ 6 \ 6$
 $4 \times 6 = 24$

We do: Joe had 4 sticker books with 6 stickers in each of them. How many stickers does she have altogether? $6 \ 6 \ 6 \ 6 \ 6$
 $4 \times 6 = 24$

You do: Lisa spend £4. Jenny spends 5 times more than Lisa. How much does Jenny spend? How much do they spend together? $4 \ 4 \ 4 \ 4 \ 4 \ 4$
 $4 \times 5 = 20$

You do: Mr and Mrs Flint's suitcases weigh 85kg in total. Mr Flint's case weighs 34kg. How much does Mrs Flint's case weigh?
 85
 $34 \quad | \quad 51$
 $80 + 5$
 $- 30 + 4$
 $50 + 1$

You do: Lisa sells 45 cups of hot chocolate in total on Saturday and Sunday. On just Saturday, she sells 33 cups. How many does she sell on Sunday?
 45
 $33 \quad | \quad 12$
 $45 - 33 =$
 $40 + 5$
 $- 30 + 3$
 $10 + 2 = 12$

You do: Remi needs to collect 35 stickers for a prize. He has collected 22 already. How many does he have left to collect?
 35
 $13 \quad | \quad 22$
 $30 + 5$
 $30 + 2$
 $10 + 3 = 13$

Appendix 3

An example of a Year Four hot task

Year 4 Action Research - Hot task (independent)

Nick has 45 flowers. He gives two of his friends 10 flowers each.
How many flowers will he have left?
 45
 $10 \ 10 \ 0$ He has 25 left.

Leah buys a ball for £12, a hat for £2 and a pair of socks for £4. She pays with a £20 note. How much change does she get?
 20
 $2 \ 4 \ 12$ He has 2 left

Kelvin has 14 cards. Janie has five times as many. How many does Janie have?
 14
 $14 \times 5 = 70$

Chloe bought an ice-cream for 52p, a packet of crisp for 35p and a can of soda for 40p. How much did she pay in total?
 $52 \ 40 \ 35$ She payed 75 in total

Carlo had some banana. She gave her mum 10 and her brother 5. She now has 15 left. How many bananas did she have to start off with?
 $10 \ 5 \ 15$ She have start off with 30 banana.

Appendix 4

Measurement of progress

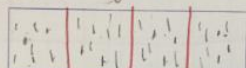
Pupils		Baseline Strategy applied?	Midpoint Strategy applied?	End Point Strategy applied?
Class 1	KA	N	Y	Y
	NA	N	P	P
	AZ	N	P	P
Class 2	NA	N	P	Y
	AW	N	P	Y
	MU	N	Y	Y
Class 3	TA	N	P	P
	OS	N	P	P
	CH	N	P	Y
Class 4	AM	N	Y	Y
	HA	N	P	P
	MA	N	N	P
Class 5	AL	N	P	Y
	RU	N	N	P
	JA	N	P	Y

Appendix 5

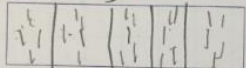
Examples of other work using bar models

Work out the answers to these questions.
Use the bar model to help you.

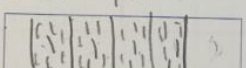
$32 \div 4 = 8$ ✓



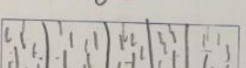
$25 \div 5 = 5$ ✓



$28 \div 4 = 7$ ✓



$40 \div 5 = 8$ ✓



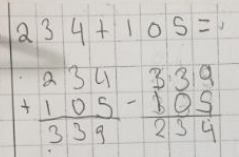
1.1. To use the inverse to check the findings of addition and subtraction questions

Steps to Success:

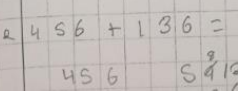
- I can use the correct method for addition and subtraction.
- I can use my resources to help me to add and subtract.
- I understand that I can use the inverse to check my answers.

CF	CA	Indep
Self Assessed	Peer Assessed	Teacher Assessed

1) $234 + 105 =$



2) $456 + 136 =$



3) $610 + 327 =$

4) $759 + 260 =$

5) $548 + 341 =$

6) $726 - 221 =$

7) $849 - 215 =$

8) $605 - 332 =$

9) $781 - 151 =$

10) $897 - 226 =$

First I had to put my numbers into column style. Next add them together. Now use inverse to check my answer. ✓